

Given an integer  $n$ , I constructed a simplicial complex  $\tau_n(\mathbb{Z})$ , called the  $n$ th Torus Complex over  $\mathbb{Z}$ , on which  $SL(n, \mathbb{Z})$  acts simplicially. The construction is analogous to that of the curve complex of a 2 dimensional torus, a complex on which  $SL(2, \mathbb{Z})$  acts. In fact,  $\tau_2(\mathbb{Z})$  is the Curve Complex of the Torus. I then found an equivalent algebraic definition of this complex that allowed me to extend the construction to rings other than  $\mathbb{Z}$ . The following table show some of the properties of  $\tau_n(R)$  that are consequences of properties of the ring  $R$ .

n	Properties of $R$	Properties of $\tau_n(R)$
2	$R = \mathbb{Z}$	Farey Graph
2	$R$ generated additively by units	link of every vertex is connected
2	$R = \mathbb{Z}[\sqrt{-n}]$ , $n = 1, 3$	$\pi_1 = \{1\}$ , not contractible for $n = -1$
3	$R = \mathbb{Z}$	connected, $\pi_1 = \{1\}$ , diameter 2
n	Euclidean	connected
$n \geq 3$	$R = \mathbb{Z}$	$\pi_{n-2}(\tau_n(\mathbb{Z})) \cong \{1\}$

The **Unoriented Torus Complex**, denoted  $\tau_n$ , is the simplicial complex whose vertices correspond to isotopy classes of essential unoriented co-dimension 1 tori. A  $k - 1$ -simplex of  $\tau_n$  ( $k - 1 \leq n$ ) is spanned by  $k$  vertices if and only if the codimension one tori to which these vertices correspond intersect transversally in exactly one codimension  $k$  torus after isotopy. This topological definition can be extended to the following algebraic definition.

The  **$n$ th Torus Complex over  $R$**  ( $R$  a commutative ring with one), denoted  $\tau_n(R)$ , is the simplicial complex whose vertices correspond to elements of an  $R$ -basis for  $R^n$  modulo sign. A  $k - 1$ -simplex of  $\tau_n(R)$  ( $k - 1 \leq n$ ) is spanned by  $k$  vertices if and only if those vertices form a subset of an  $R$ -basis of  $R^n$ .

**Theorem 1.**  $\tau_3(\mathbb{Z})$  is simply connected and  $SL(3, \mathbb{Z})$  acts on  $\tau_3(\mathbb{Z})$  cocompactly (but not properly). Taking the fundamental group of the complex of groups obtained by labeling cells in the quotient by their stabilizer groups yields the following presentation for  $SL(3, \mathbb{Z})$ .

$$\langle a, b, c | a^4, b^6, c^2, a^2b^3, (a^2cab)^2, (cabcb^2)^3, (ac)^3, \\ ab(cab)(ab)^{-1}(cab)^{-1}, ba(cba)(ba)^{-1}(cba)^{-1} \rangle$$